№71
$$\begin{cases} U_t = 4U_{xx} + U \\ U_x(0,t) = 0 \\ U(1,t) = 0 \\ U(x,0) = 2\cos(2\pi x)\cos\left(\frac{3\pi}{2}x\right) \end{cases}$$

$$U(x,t) = XT'$$

$$XT' = 4X''T - XT' \,\Big|\, \frac{1}{4XT}$$

$$\frac{T''}{4T} - \frac{1}{4} = \frac{X''}{X} = -\lambda^2$$

$$\frac{T'}{4T} - \frac{1}{4} + \lambda^2 = 0$$

$$X_n = A_n \sin\lambda_n x + B_n \cos\lambda_n x$$

$$X_n' = A_n\lambda_n \cos\lambda_n x - B_n\lambda_n \sin\lambda_n x$$

$$A_n = 0 \Rightarrow X_n' = -B_n\lambda_n \sin\lambda_n x \Rightarrow \sin\lambda_n = 0 \Rightarrow$$

$$\Rightarrow \lambda_n = \pi n,\ n \in \mathbb{Z}$$

$$\{a_n = B_n\cos\lambda_n x,\ \lambda_n = \pi n,\ n \in \mathbb{Z}\}$$

$$\frac{T'}{4T} + \lambda^2 - \frac{1}{4} = 0 \Rightarrow T_n = C_n e^{(1-4\lambda_n^2)t}$$

$$U(x,t) = \sum_{n=1}^{\infty} C_n e^{(1-4\lambda_n^2)t}\cos(\lambda_n x)$$

$$U(x,0) = \sum_{n=1}^{\infty} C_n \cos \pi n x = 2\cos 2\pi x \cos\frac{3\pi}{2}x$$

$C_n$ - ?

$\underline{N72}$
$$\begin{cases} u_t = u_{xx} + u \\ u_x(0,t) = u_x(\frac{\pi}{2}, t) = 0 \\ u(x,0) = 8\cos^4 x \end{cases}$$

$$u(x,t) = X(x)\,T(x)$$
$$XT' = X''T + XT$$
$$\frac{T'}{T} = \frac{X'}{X} + 1$$
$$\frac{T'}{T} - 1 = \frac{X''}{X} = \lambda^2$$
$$X_n = A\sin\lambda_n x + B\cos\lambda_n x$$
$$X'' + \lambda^2 X = 0$$
$$\frac{T'}{T} + \lambda^2 - 1 = 0$$
$$X_n = A\sin\lambda_n x + B\cos\lambda_n x$$
$$X_n' = A_n\cos\lambda_n x - B\lambda_n\sin\lambda_n x$$
$$X_n'(0) = 0; \quad X_n'\left(\frac{\pi}{2}\right) = 0 \Rightarrow \left[\begin{matrix} \lambda_n = 0 \\ 0, \lambda = 0, B \neq 0 \end{matrix}\right.$$
$$X_n' = -B\lambda_n\sin\lambda_n x$$
$$\sin\lambda_n\frac{\pi}{2} = 0$$
$$\lambda_n x = \pi n$$

$$\lambda_n = \frac{\pi \cdot n}{\pi/2} = 2n$$
$$\begin{cases} \lambda_n = 2n \\ X_n = B_n\cos(\lambda_n x) \end{cases}$$
$$\frac{T'}{T} + \lambda^2 - 1 = 0$$
$$T' - T + \lambda^2 T = 0$$
$$T_n(t) = C_n e^{t - \lambda_n^2 t}$$
$$u(x,t) = \sum_{n=1}^{\infty} C_n B_n e^{t-\lambda_n^2 t}\cos(\lambda_n x) = \sum_{n=1}^{\infty} C_n e^{t-\lambda_n^2 t}\cos(\lambda_n x)$$
$$\cos^4(x) = \frac{1}{8}(4\cos(2x) + \cos(4x) + 3) = \frac{1}{2}\cos 2x +$$
$$+ \frac{1}{8}\cos^4 x + \frac{3}{8} \Rightarrow$$

$u(x,t) = ?$

$$8\cos^4(x) = 4\cos(2x) + \cos(4x) + 3$$

$$u(x,t) = 3 + 4e^{-3t}\cos 2x + e^{-15t}\cos 4x$$

№74

$$\begin{cases} U_t = 5U_{xx} + U_x - U \\ U_x(0,t) = 0 \\ U(1,t) = 0 \\ U(x,0) = 3 \end{cases}$$

$$U(x,t) = \sum X_n(x) T_n(t)$$

$$X_n T_n' = 5 X_n'' T_n + X_n' T_n - X_n T_n \Big| \cdot \frac{1}{X_n T_n}$$

$$\frac{T_n'}{T_n} = \frac{5X_n''}{X_n} + \frac{X_n'}{X_n} - 1$$

$$\frac{T_n'}{T_n} + 1 = \frac{5X_n''}{X_n} + \frac{X_n'}{X_n}$$

$$\frac{T_n'}{5T_n} + \frac{1}{5} = \frac{X_n''}{X_n} + \frac{X_n'}{5X_n} = -\lambda_n^2$$

$$\begin{cases} X_n'' + \frac{1}{5}X_n' = -\lambda_n^2 X_n \\ X_n'(0) = 0 \\ X_n(1) = 0 \end{cases}$$

$$K^2 + \frac{1}{5}K + \lambda^2 = 0$$

$$D = \frac{1}{25} - 4\lambda^2 \Rightarrow K_{1,2} = \frac{-\frac{1}{5} \pm \sqrt{\frac{1}{25} - 4\lambda^2}}{2} = -\frac{1}{10} \pm \sqrt{\frac{1}{100} - \lambda^2} = -\frac{1}{10} \pm \omega i$$

$$X_n(x) = C_1 e^{(-\frac{1}{10} + \omega i)x} + C_2 e^{(-\frac{1}{10} - \omega i)x} = e^{-\frac{x}{10}}(A\sin\omega x + B\cos\omega x) \; ; \; \omega^2 = \lambda^2 - \frac{1}{100}$$

$$\begin{cases} X_n'(0) = 0 = -\frac{1}{10}B + A\omega \\ X_n(1) = 0 = e^{-\frac{1}{10}}(A\sin\omega + B\cos\omega) \end{cases}$$

$$\left(-\frac{B}{10\omega}\sin\omega + B\cos\omega\right)e^{-\frac{1}{10}} = 0$$

$$B\left(\cos\omega - \frac{1}{10\omega}\sin\omega\right) = 0$$

$$B \neq 0, \Rightarrow \cos\omega = \frac{1}{10\omega}\sin\omega$$

$$\operatorname{tg}\omega = 10\omega$$

$$\lambda_n = \frac{\omega_n}{\sqrt{\omega_n^2 + \frac{1}{100}}}$$

$$X_n(x) = e^{-\frac{x}{10}} B\left(-\frac{1}{10\omega_n}\sin\omega_n x + \cos\omega_n x\right)$$

$$\frac{T_n'}{5T_n} + \frac{1}{5} = -\lambda_n^2$$

$$T_n' + T_n = -5\lambda_n^2 T_n$$

$$T_n' + T_n(1 + 5\lambda_n^2) = 0$$

$$T_n = C_n e^{-(1+5\lambda_n^2)} = C_n e^{-(1+5\omega_n^2+\frac{1}{20})x}$$

$$U(x,t) = \sum_{n=1}^{\infty} X_n(x) T_n(t)$$

$$U(x,0) = \sum_{n=1}^{\infty} C_n X_n(x) = 3$$

$$C_n = 2\int_0^1 3 \cdot X_n(x)\,dx$$

$$u(x,t) = \;?$$